nag_ode_ivp_rk_onestep (d02pdc)

1. Purpose

nag_ode_ivp_rk_onestep (d02pdc) is a one-step function for solving the initial value problem for a first order system of ordinary differential equations using Runge-Kutta methods.

2. Specification

```
#include <nag.h>
#include <nagd02.h>

void nag_ode_ivp_rk_1step(Integer neq, void (*f)(Integer neq, double t, double y[],
                                      double yp[], Nag_User *comm),
           double *tnow, double ynow[],
           double ypnow[], Nag_ODE_RK *opt, Nag_User *comm,
           NagError *fail)
```

3. Description

This function and its associated functions (nag_ode_ivp_rk_setup (d02pvc), nag_ode_ivp_rk_reset_tend (d02pwc), nag_ode_ivp_rk_interp (d02pxc), nag_ode_ivp_rk_errass (d02pzc)) solve the initial value problem for a first order system of ordinary differential equations. The functions, based on Runge-Kutta methods and derived from RKSUITE (Brankin *et al*, 1991), integrate

$$y' = f(t, y) \qquad \text{given} \qquad y(t_0) = y_0$$

where y is the vector of **neq** solution components and t is the independent variable.

This function is designed to be used in complicated tasks when solving systems of ordinary differential equations. You must first call nag_ode_ivp_rk_setup (d02pvc) to specify the problem and how it is to be solved. Thereafter you (repeatedly) call nag_ode_ivp_rk_onestep to take one integration step at a time from **tstart** in the direction of **tend** (as specified in nag_ode_ivp_rk_setup (d02pvc)). In this manner nag_ode_ivp_rk_onestep returns an approximation to the solution **ynow** and its derivative **ypnow** at successive points **tnow**. If nag_ode_ivp_rk_onestep encounters some difficulty in taking a step, the integration is not advanced and the routine returns with the same values of **tnow**, **ynow** and **ypnow** as returned on the previous successful step. nag_ode_ivp_rk_onestep tries to advance the integration as far as possible subject to passing the test on the local error and not going past **tend**. In the call to nag_ode_ivp_rk_setup (d02pvc) you can specify either the first step size for nag_ode_ivp_rk_onestep to attempt or that it compute automatically an appropriate value. Thereafter nag_ode_ivp_rk_onestep estimates an appropriate step size for its next step. This value and other details of the integration can be obtained after any call to nag_ode_ivp_rk_onestep by examining the contents of the structure **opt**, see Section 4. The local error is controlled at every step as specified in nag_ode_ivp_rk_setup (d02pvc). If you wish to assess the true error, you must set **errass** = **Nag_ErrorAssess_on** in the call to nag_ode_ivp_rk_setup (d02pvc). This assessment can be obtained after any call to nag_ode_ivp_rk_onestep by a call to the subroutine nag_ode_ivp_rk_errass (d02pzc).

If you want answers at specific points there are two ways to proceed:

The more efficient way is to step past the point where a solution is desired, and then call nag_ode_ivp_rk_interp (d02pxc) to get an answer there. Within the span of the current step, you can get all the answers you want at very little cost by repeated calls to nag_ode_ivp_rk_interp (d02pxc). This is very valuable when you want to find where something happens, e.g., where a particular solution component vanishes. You cannot proceed in this way with **method** = **Nag_RK_7_8**.

The other way to get an answer at a specific point is to set **tend** to this value and integrate to **tend**. nag_ode_ivp_rk_onestep will not step past **tend**, so when a step would carry it past, it will reduce the step size so as to produce an answer at **tend** exactly. After getting an answer there (**tnow** = **tend**), you can reset **tend** to the next point where you want an answer, and repeat. **tend** could be reset by a call to nag_ode_ivp_rk_setup (d02pvc), but you should not do this. You should use nag_ode_ivp_rk_reset_tend (d02pwc) because it is both easier to use and much more efficient. This way of getting answers at specific points can be used with any of the available methods, but it is the only way with **method** = **Nag_RK_7_8**. It can be inefficient. Should this be the case, the code will bring the matter to your attention.

4. Parameters

neq

Input: the number of ordinary differential equations in the system to be solved.
Constraint: **neq** ≥ 1.

f

This function must evaluate the first derivatives y_i' (that is the functions f_i) for given values of the arguments t, y_i.

```
void f (Integer neq, double t, double y[], double yp[], Nag_User *comm)
```

neq

Input: the number of differential equations.

t

Input: the current value of the independent variable, t.

y[neq]

Input: the current values of the dependent variables, y_i for $i = 1, 2, \ldots,$**neq**.

yp[neq]

Output: the values of f_i for $i = 1, 2, \ldots,$**neq**.

comm

Input/Output: pointer to a structure of type Nag_User with the following member:

p - Pointer
Input/Output: The pointer **comm->p** should be cast to the required type, e.g. `struct user *s = (struct user *)comm->p`, to obtain the original object's address with appropriate type. (See the argument **comm** below.)

tnow

Output: the value of the independent variable t at which a solution has been computed.

ynow[neq]

Output: an approximation to the solution at **tnow**. The local error of the step to **tnow** was no greater than permitted by the specified tolerances (see nag_ode_ivp_rk_setup (d02pvc)).

ypnow[neq]

Output: an approximation to the derivative of the solution at **tnow**.

opt

Input: pointer to a structure of type Nag_ODE_RK as initialised by the setup function nag_ode_ivp_rk_setup (d02pvc).
Output: the following structure members hold information as follows:

totfcn - Integer
The total number of evaluations of f used in the primary integration so far; this does not include evaluations of f for the secondary integration specified by a prior call to nag_ode_ivp_rk_setup (d02pvc) with errass = **Nag_ErrorAssess_on**.

stpcst - Integer
The cost in terms of number of evaluations of f of a typical step with the method being used for the integration. The method is specified by the parameter method in a prior call to nag_ode_ivp_rk_setup (d02pvc).

waste - double
The number of attempted steps that failed to meet the local error requirement divided by the total number of steps attempted so far in the integration. A "large" fraction indicates that

the integrator is having trouble with the problem being solved. This can happen when the problem is “stiff” and also when the solution has discontinuities in a low order derivative.

stpsok - Integer
The number of accepted steps.

hnext - double
The step size the integrator plans to use for the next step.

comm
Input/Output: pointer to a structure of type Nag_User with the following member:

p - Pointer
Input/Output: The pointer **p**, of type Pointer, allows the user to communicate information to and from the user-defined function **f()**. An object of the required type should be declared by the user, e.g. a structure, and its address assigned to the pointer **p** by means of a cast to Pointer in the calling program, e.g. `comm.p = (Pointer)&s`. The type pointer will be `void *` with a C compiler that defines `void *` and `char *` otherwise.

fail
The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_PREV_CALL
The previous call to a function had resulted in a severe error. You must call nag_ode_ivp_rk_setup (d02pvc) to start another problem.

NE_NO_SETUP
The setup function nag_ode_ivp_rk_setup (d02pvc) has not been called.

NE_RK_INVALID_CALL
The function to be called as specified in the setup routine nag_ode_ivp_rk_setup (d02pvc) was nag_ode_ivp_rk_range (d02pcc). However the actual call was made to nag_ode_ivp_rk_onestep. This is not permitted.

NE_PREV_CALL_INI
The previous call to the function nag_ode_ivp_rk_onestep had resulted in a severe error. You must call nag_ode_ivp_rk_setup (d02pvc) to start another problem.

NE_NEQ
The value of neq supplied is not the same as that given to the setup function nag_ode_ivp_rk_setup (d02pvc). **neq** = ⟨*value*⟩ but the value given to nag_ode_ivp_rk_setup (d02pvc) was ⟨*value*⟩.

NE_RK_PDC_TEND
tend (= ⟨*value*⟩) has been reached already. To integrate further with same problem the function nag_ode_ivp_rk_reset_tend (d02pwc) must be called with a new value of **tend**.

NE_RK_PDC_STEP
In order to satisfy the error requirements nag_ode_ivp_rk_onestep would have to use a step size of ⟨*value*⟩ at current **t** = ⟨*value*⟩. This is too small for the ***machine precision***.

NW_RK_TOO_MANY
Approximately ⟨*value*⟩ function evaluations have been used to compute the solution since the integration started or since this message was last printed.

NE_RK_PDC_GLOBAL_ERROR_T
The global error assessment may not be reliable for t past **tnow**. **tnow** = ⟨*value*⟩.

NE_RK_PDC_GLOBAL_ERROR_S
The global error assessment algorithm failed at the start of the integration.

NE_RK_PDC_POINTS

More than 100 output points have been obtained by integrating to **tend**. They have been sufficiently close to one another that the efficiency of the integration has been degraded. It would probably be (much) more efficient to obtain output by interpolating with nag_ode_ivp_rk_interp (d02pxc) (after changing to **method** = **Nag_RK_4_5** if you are using **method** = **Nag_RK_7_8**).

NE_STIFF_PROBLEM

The problem appears to be stiff.

NE_MEMORY_FREED

Internally allocated memory has been freed by a call to nag_ode_ivp_rk_free (d02ppc) without a subsequent call to the set up function nag_ode_ivp_rk_setup (d02pvc).

6. Further Comments

If nag_ode_ivp_rk_onestep returns with fail.code = **NE_RK_PDC_STEP** and the accuracy specified by **tol** and **thres** is really required then you should consider whether there is a more fundamental difficulty. For example, the solution may contain a singularity. In such a region the solution components will usually be of a large magnitude. Successive output values of **ynow** should be monitored with the aim of trapping the solution before the singularity. In any case numerical integration cannot be continued through a singularity, and analytical treatment may be necessary.

If nag_ode_ivp_rk_onestep returns with a non-trivial value of fail.code (i.e., those not related to an invalid call) then performance statistics are available by examining the structure **opt** (see Section 4). Furthermore if **errass** was set to **Nag_ErrorAssess_on** then global error assessment is available by a call to the function nag_ode_ivp_rk_errass (d02pzc). The approximate extra number of evaluations of f used is given by $2 \times$ **stpsok** $\times$ **stpcst** for method **Nag_RK_4_5** or **Nag_RK_7_8** and $3 \times$ **stpsok** $\times$ **stpcst** for method = **Nag_RK_2_3**.

After a failure with fail.code = **NE_RK_PDC_STEP**, **NE_RK_PDC_GLOBAL_ERROR_T** or **NE_RK_PDC_GLOBAL_ERROR_S** the diagnostic function nag_ode_ivp_rk_errass (d02pzc) may be called only once.

If nag_ode_ivp_rk_onestep returns with fail.code = **NE_STIFF_PROBLEM** then it is advisable to change to another code more suited to the solution of stiff problems. nag_ode_ivp_rk_onestep will not return with fail.code = **NE_STIFF_PROBLEM** if the problem is actually stiff but it is estimated that integration can be completed using less function evaluations than already computed.

6.1. Accuracy

The accuracy of integration is determined by the parameters **tol** and **thres** in a prior call to nag_ode_ivp_rk_setup (d02pvc). Note that only the local error at each step is controlled by these parameters. The error estimates obtained are not strict bounds but are usually reliable over one step. Over a number of steps the overall error may accumulate in various ways, depending on the properties of the differential system.

6.2. References

Brankin R W, Gladwell I and Shampine L F (1991) *RKSUITE: a suite of Runge-Kutta codes for the initial value problem for ODEs* SoftReport 91-S1, Department of Mathematics, Southern Methodist University, Dallas, TX 75275, U.S.A.

7. See Also

nag_ode_ivp_rk_setup (d02pvc)
nag_ode_ivp_rk_reset_tend (d02pwc)
nag_ode_ivp_rk_interp (d02pxc)
nag_ode_ivp_rk_errass (d02pzc)

8. Example

We solve the equation

$$y'' = -y, \qquad y(0) = 0, y'(0) = 1$$

reposed as

$$y_1' = y_2 \qquad y_2' = -y_1$$

over the range $[0, 2\pi]$ with initial conditions $y_1 = 0.0$ and $y_2 = 1.0$. We use relative error control with threshold values of 1.0e−8 for each solution component and print the solution at each integration step across the range. We use a medium order Runge-Kutta method (**method** = **Nag_RK_4_5**) with tolerances **tol** = 1.0e−4 and **tol** = 1.0e−5 in turn so that we may compare the solutions. The value of π is obtained by using X01AAC.

See also the example programs for nag_ode_ivp_rk_reset_tend (d02pwc) and nag_ode_ivp_rk_interp (d02pxc).

8.1. Program Text

```
/* nag_ode_ivp_rk_onestep(d02pdc) Example Program
 *
 * Copyright 1994 Numerical Algorithms Group.
 *
 * Mark 3, 1994.
 *
 */

#include <nag.h>
#include <math.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagd02.h>
#include <nagx01.h>

#ifdef NAG_PROTO
static void f(Integer neq, double t1, double y[], double yp[], Nag_User *comm);
#else
static void f();
#endif

#define NEQ 2
#define ZERO 0.0
#define ONE 1.0
#define TWO 2.0
#define FOUR 4.0

main()
{
  Integer neq;
  Nag_RK_method method;
  double hstart, pi, tnow, tend;
  double  tol, tstart;
  Integer i;
  double thres[NEQ], ynow[NEQ], ypnow[NEQ], ystart[NEQ];
  Nag_ErrorAssess errass;
  Nag_ODE_RK opt;
  Nag_User comm;

  Vprintf("d02pdc Example Program Results\n");

  /*  Set initial conditions and input for d02pvc */
  neq = NEQ;
  pi = X01AAC;
  tstart = ZERO;
  ystart[0] = ZERO;
  ystart[1] = ONE;
  tend = TWO*pi;
  for (i=0; i<neq; i++)
    thres[i] = 1.0e-8;
  errass = Nag_ErrorAssess_off;
  hstart = ZERO;
  method  = Nag_RK_4_5;

  for (i=1; i<=2; i++)
    {
      if (i==1) tol = 1.0e-4;
      if (i==2) tol = 1.0e-5;
      d02pvc(neq, tstart, ystart, tend, tol, thres, method,
             Nag_RK_onestep, errass, hstart, &opt, NAGERR_DEFAULT);

      Vprintf("\nCalculation with tol = %8.1e\n\n",tol);
      Vprintf ("    t          y1          y2\n\n");
      Vprintf("%8.3f   %8.3f   %8.3f\n", tstart, ystart[0], ystart[1]);
      do
        {

          d02pdc(neq, f, &tnow, ynow, ypnow, &opt, &comm,
                 NAGERR_DEFAULT);
          Vprintf("%8.3f   %8.3f   %8.3f\n", tnow, ynow[0], ynow[1]);
        } while (tnow<tend);

      Vprintf("\nCost of the integration in evaluations of f is %ld\n\n",
              opt.totfcn);
      d02ppc(&opt);
    }
  exit(EXIT_SUCCESS);
}
#ifdef NAG_PROTO
static void f(Integer neq, double t, double y[], double yp[], Nag_User *comm)
#else
     static void f(neq, t, y, yp, comm)
     Integer neq;
     double t;
     double y[], yp[];
     Nag_User *comm;
#endif

{
  yp[0] = y[1];
  yp[1] = -y[0];
}
```

8.2. Program Data

None.

8.3. Program Results

```
d02pdc Example Program Results

Calculation with tol =  1.0e-04

    t          y1          y2

   0.000      0.000      1.000
   0.785      0.707      0.707
   1.519      0.999      0.051
   2.282      0.757     -0.653
   2.911      0.229     -0.974
   3.706     -0.535     -0.845
   4.364     -0.940     -0.341
   5.320     -0.821      0.571
   5.802     -0.463      0.886
   6.283      0.000      1.000

Cost of the integration in evaluations of f is 78
```

```
Calculation with tol =  1.0e-05

    t          y1         y2

   0.000      0.000      1.000
   0.393      0.383      0.924
   0.785      0.707      0.707
   1.416      0.988      0.154
   1.870      0.956     -0.294
   2.204      0.806     -0.592
   2.761      0.371     -0.929
   3.230     -0.088     -0.996
   3.587     -0.430     -0.903
   4.022     -0.771     -0.637
   4.641     -0.997     -0.072
   5.152     -0.905      0.426
   5.521     -0.690      0.724
   5.902     -0.372      0.928
   6.283      0.000      1.000

Cost of the integration in evaluations of f is 118
```